IT-312 M.Ali R# 20221

Linear Algebra

2022

Short Questions:

i) Define reduced row echelon form and give 3 examples.

Answer:

Reduced Row Echelon Form:

An echelon matrix in which each pivot (first non-zero entry of a row) is 1 and every other entry of the pivot's column is zero, it is said to be in row reduced echelon form.

Example:

i) $\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, ii) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 7/5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii) Find a row operation and the corresponding elementary matrix that will restore the elementary matrix $\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}$ to the identity matrix.

Solution:

Let $A = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}$

Using row operations,

$R_1 - 1/2 R_2$

$\overset{R}{\sim} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Ans

iii) Confirm the identity without evaluating the determinant directly.

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Solution:

$$L.H.S = \begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$R_1 + R_2$

$$= \begin{vmatrix} a_1 + a_1t + b_1t + b_1 & a_2 + a_2t + b_2t + b_2 & a_3 + a_3t + b_3t + b_3 \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1(1+t) + b_1(1+t) & a_2(1+t) + b_2(1+t) & a_3(1+t) + b_3(1+t) \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} (a_1+b_1)(1+t) & (a_2+b_2)(1+t) & (a_3+b_3)(1+t) \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Taking common $(1+t)$ from R_1

$$= (1+t) \begin{vmatrix} a_1 + b_1 & a_2 + b_2 & a_3 + b_3 \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$R_1 - R_2$

$$= (1+t) \begin{vmatrix} a_1 + \cancel{b_1} - a_1t - \cancel{b_1} & a_2 + \cancel{b_2} - a_2t - \cancel{b_2} & a_3 + \cancel{b_3} - a_3t - \cancel{b_3} \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1+t) \begin{vmatrix} a_1(1-t) & a_2(1-t) & a_3(1-t) \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Taking common $(1-t)$ from R_1

$$= (1+t)(1-t) \begin{vmatrix} a_1 & a_2 & a_3 \\ a_1t + b_1 & a_2t + b_2 & a_3t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$R_2 - tR_1$

$$= (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ a_1t + b_1 - a_1t & a_2t + b_2 - a_2t & a_3t + b_3 - a_3t \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= R.H.S$

Hence Proved

iv) Determine whether the polynomials $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$ in P_2 are linearly dependent.

Solutions

Let $a, b, c \in R$, so

$$a(2 - x + 4x^2) + b(3 + 6x + 2x^2) + c(2 + 10x - 4x^2) = 0$$

$$2a - ax + 4ax^2 + 3b + 6bx + 2bx^2 + 2c + 10cx - 4cx^2 = 0$$

$$(4a + 2b - 4c)x^2 + (-a + 6b + 10c)x^2 + (2a + 3b + 2c) = 0$$

$$\Rightarrow \quad \begin{aligned} 4a + 2b - 4c &= 0 \\ -a + 6b + 10c &= 0 \\ 2a + 3b + 2c &= 0 \end{aligned}$$

In Homogeneous linear system, if Rank = variables, it is linearly independent, if Rank < variables, it is linearly dependent, if Rank > variables, the system is inconsistent. So, we have to find the

Rank of system.

Augumented Matrix.

$$\cancel{\begin{bmatrix} 4a & 2 \\ -a & \\ 2a & \end{bmatrix}}$$

$$\sim \begin{bmatrix} 4 & 2 & -4 \\ -1 & 6 & 10 \\ 2 & 3 & 2 \end{bmatrix}$$

$$\frac{1}{4}R_1 \quad \sim \begin{bmatrix} 1 & 1/2 & -1 \\ -1 & 6 & 10 \\ 2 & 3 & 2 \end{bmatrix}.$$

$R_2 + R_1 \quad ; \quad R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1/2 & -1 \\ 0 & 13/2 & 9 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\frac{2}{13}R_2 \quad \begin{bmatrix} 1 & 1/2 & -1 \\ 0 & 1 & 18/13 \\ 0 & 2 & 4 \end{bmatrix}$$

$R_1 - \frac{1}{2}R_2 \quad ; \quad R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 0 & -22/13 \\ 0 & 1 & 18/13 \\ 0 & 0 & 16/13 \end{bmatrix}$$

Rank of A = no. unknown variables (a, b, c)

So, it is linearly independent, because $a = b = c = 0$.

Ans

v) Discuss how the rank of $A = \begin{bmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{bmatrix}$ varies with t.

Solution:

Given:

$$A = \begin{bmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{bmatrix}$$

$R_2 - R_1$; $R_3 - tR_1$

$$\sim \begin{bmatrix} 1 & 1 & t \\ 0 & t-1 & 1-t \\ 0 & 1-t & 1-t^2 \end{bmatrix}$$

$R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & t \\ 0 & t-1 & 1-t \\ 0 & \cancel{1}-\cancel{t}+\cancel{t}-\cancel{1} & 1-t^2+1-t \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & t \\ 0 & t-1 & 1-t \\ 0 & 0 & 2-t^2-t \end{bmatrix}$$

Rank of $A = 3$ for $2-t^2-t \neq 0$

Rank of $A = 2$ for $2-t^2-t = 0$

$2-t^2-t = 0$

$t^2+t-2 = 0$

$t^2+2t-t-2 = 0$

$t(t+2)-1(t+2) = 0$

$(t+2)(t-1) = 0$

$t = 1$, $t = -2$

So, Rank of $A = 2$ for $t = 1, -2$

Ans

vi) The eigenvalues of a matrix A are the same as the eigenvalues of the reduced row echelon form of A. Determine whether the statement is true or false, and justify your answer.

Answer:

The statement is false because the eigenvalues of a matrix are not equal to those of its reduced row echelon form.

This is because row reduction changes the determinant of a matrix, and the eigenvalues are the solutions of the characteristic equation $\det(A - \lambda I) = 0$.

eg

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1 \\ -1 & 3-\lambda \end{vmatrix}$$

$$= (1-\lambda)(3-\lambda) + 1(+1)$$

$$= 3 - \lambda - 3\lambda + \lambda^2 + 1$$

$$= \lambda^2 - 4\lambda + 4$$

$$= \lambda^2 - 2\lambda - 2\lambda + 4$$

$$= \lambda(\lambda - 2) - 2(\lambda - 2)$$

$$0 = (\lambda - 2)(\lambda - 2)$$

$$\Rightarrow \lambda = 2$$

Row Reduced Echelon Form

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$= (1-\lambda)(1-\lambda)$$

$$= 1 - \lambda - \lambda + \lambda^2$$

$$= \lambda(\lambda - 1) - 1(\lambda - 1)$$

$$0 = (\lambda - 1)(\lambda - 1)$$

$$\Rightarrow \lambda = 1$$

Long Questions

Q.2 Solve the following system of non-linear equations for x, y, z:

$$x^2 + y^2 + z^2 = 6$$
$$x^2 - y^2 + 2z^2 = 2$$
$$2x^2 + y^2 - z^2 = 3$$

Solution:

Let $a = x^2$, $b = y^2$, $c = z^2$, so

$$\cancel{a^2 + b^2 + c^2 = 6}$$

$$a + b + c = 6$$
$$a - b + 2c = 2$$
$$2a + b - c = 3$$

Augumented Matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

By Gaussian Elimination method,

$R_2 - R_1$; $R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -4 \\ 0 & -1 & -3 & -9 \end{array}\right]$$

$-1/2 R_2$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & -1 & -3 & -9 \end{array}\right]$$

$R_3 + R_2$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & -7/2 & -7 \end{array}\right]$$

$$\Rightarrow \quad a + b + c = 6 \quad -(i)$$

$$b - \frac{c}{2} = 2 \quad -(ii)$$

$$-\frac{7}{2} c = -7 \quad -(iii)$$

From (iii)

$$+\frac{7}{2} c = +7$$

$$\cancel{7} c = \cancel{14}\, 2 \quad \Rightarrow \quad c = 2$$

Put the value of c in (ii)

$$\cancel{2} \cancel{-} \frac{c}{\cancel{2}} \cancel{= 2}$$

$$b - \frac{\cancel{2}^{1}}{\cancel{2}_{1}} = 2$$

$$b = 2 + 1$$

$$\boxed{b = 3}$$

From (i)

$$a + 3 + 2 = 6$$

$$\boxed{a = 1}$$

Now,

$$a = x^2 = 1$$
$$b = y^2 = 3$$
$$c = z^2 = 2$$

$x = 1, \quad y = \sqrt{3}, \quad z = \sqrt{2}$

Ans

Q.3 Find the coordinate vector of A relative to the basis $S = \{A_1, A_2, A_3, A_4\}$

$A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$; $A_1 = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$,

$A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Let $a, b, c, d \in R$, linear combination:

$$\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} = a\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$
$$+ d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -a & a \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} b & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -a+b & a+b \\ c & d \end{bmatrix}$$

$\Rightarrow$ $2 = -a+b$ —(i)

$0 = a+b$ —(ii)

$\boxed{-1 = c}$

$\boxed{d = 3}$

Adding (i) and (ii)

$$2+0 = -\cancel{a}+b+\cancel{a}+b$$

$$\cancel{2} = \cancel{2}b$$

$$\boxed{b = 1}$$

Put in (i)

$$2 = -a+1$$

$$2-1 = -a$$

$$\boxed{a = -1}$$

So, the co-ordinate vector of A is $[-1, 1, -1, 3]$

Ans

Q.4 Find a matrix P that diagonalizes A, and compute $P^{-1}AP$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

To find P, we need to find eigen-vectors, so

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

Expand from R_1

$$(1-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)(1-\lambda) - 1\right] = 0$$

$$(1-\lambda)\left[\cancel{1} - \lambda - \lambda + \lambda^2 - \cancel{1}\right] = 0$$

$$(1-\lambda)\left[\lambda^2 - 2\lambda\right] = 0$$

$$\lambda^2 - 2\lambda - \lambda^3 + 2\lambda^2 = 0$$

$$-\lambda^3 + 3\lambda^2 - 2\lambda = 0$$

$$-\lambda(\lambda^2 - 3\lambda + 2) = 0$$

$-\lambda = 0$

$\lambda = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda - 2) - 1(\lambda - 2)$

$(\lambda - 2)(\lambda - 1) = 0$

$\lambda = 1, \lambda = 2$

Eigen values are $\lambda = 0, 1, 2$

For $\lambda = 0$,

$$\begin{bmatrix} 1-0 & 0 & 0 \\ 0 & 1-0 & 1 \\ 0 & 1 & 1-0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank of ~~A~~ $A - \lambda I$ $<$ no of unknown variables

$\Rightarrow \quad x = 0 \quad —(i)$

$y + z = 0 \quad —(ii)$

z is a free variable, so let

$z = t$

From (ii)

$y + t = 0 \quad \Rightarrow \quad y = -t$

So Eigen vector for $\lambda = 0$,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \cdot t$$

For $t = 1$,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda = 1$,

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 1-1 & 1 \\ 0 & 1 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ Rank of $A - \lambda I$

$\Rightarrow \quad x$ is free variable, let $x = t$

$y = 0$

$z = 0$

So, eigen vector for $\lambda = 1$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot t$

For $t=1$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

For $\lambda = 2$,

$$\begin{bmatrix} 1-2 & 0 & 0 \\ 0 & 1-2 & 1 \\ 0 & 1 & 1-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ Rank $A - \lambda I$ > no of unknown variables

$\Rightarrow$

$-x = 0 \Rightarrow \boxed{x = 0}$

$-y + z = 0 \quad —(1)$

z is a free variable, so let $\boxed{z = t}$,

From (1)

$-y + t = 0$

$\boxed{y = t}$

So, eigen vector for $\lambda = 2$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cdot t$

For $t = 1$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

The eigen vectors are: $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

So, $P = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

To find P^{-1}

Append I_3 with P

$$\sim \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

R_{12}

$$\sim \left[\begin{array}{ccc|ccc} -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$-R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 - R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 \end{array}\right]$$

$\frac{1}{2} R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right]$$

$R_1 + R_3$

$$\overset{R}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right]$$

So,

$$P^{-1} = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 & 0 \\ \cancel{-1\times1+1\times1} & 0 & 1\times1+1\times1 \\ \cancel{-1\times1+1\times1} & 0 & 1\times1+1\times1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & -1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & 0 & -1/\cancel{2}\times\cancel{2} + \frac{1}{\cancel{2}}\times\cancel{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\cancel{2}}\times\cancel{2} + \frac{1}{\cancel{2}}\times\cancel{2} \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = D$$

(Diagonal Matrix)

P diagonalizes the matrix A into diagonal matrix D. Answer

(Best of Luck)