18/July/2023 Lec#02 Tuesday

linear algebra

Algebra:- is the study of variables and the rules for manipulating these variables in formulas.

linear algebra:-
linear algebra is a branch of mathematics concerning linear equations linear maps and their representations in vector space and through matrices.

ChP # 02

"System of linear Equations"

linear Equation:-
An equation in which variables' highest power is 1.

1) $x+2=3$.
2) $x+y=5$

* General linear equation

$$a_1x_1 + a_2x_2 + \text{———} + a_nx_n = b_n$$

System of linear equation → (linear system)

A system (group) of two or more linear equations involving same variables is called system of linear equations.

Example :-

(i)
$$2x_1 + x_2 + 3x_3 = 7$$
$$-4x_1 + x_3 = 6$$
$$7x_1 - x_2 + 4x_3 = 2$$

(ii)
$$x + 2y = 7$$
$$x - 3y + z = 8$$

Solution of linear system :-

$$2x_1 - x_2 + 1.5x_3 = 8 \quad \text{——} (1)$$
$$x_1 - 4x_3 = -7 \quad \text{——} (2)$$

its sol set is.

$$(x_1, x_2, x_3) = (5, 6.5, 3)$$

Put these values in (1)

$$2(5) - (6.5) + 1.5(3) = 8$$
$$10 - 6.5 + 3.5 = 8$$

$8 = 8$

Put these values in (2)

$5 - 4(3) = -7$

$5 - 12 = -7$

$-7 = -7$

satisified

* General linear system

$a_{11}x_1 + a_{12} + \text{———} + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22} + \text{———} + a_{2n}x_n + b_2$

$\vdots \qquad\qquad \vdots$

$a_{m1}x_1 + a_{m2}x_2 + \text{———} + a_{mn}x_n = b_m$

So this is a general linear system of 'm' equations in 'n' variables.

Solution of linear system :-

A system of linear system has either

i) No solution ⟶ inconsistent system
ii) Exactly one solution (trivial solution)
iii) infinitly many solution (no trival sol) } ⟶ inconsistent

(2) A system of linear equation has no solution ?

$x_1 - 2x_2 = -1$ — ①
$-x_1 + 2x_2 = 3$ — ②

Put $x_1 = 0$ in ①

$-2x_2 = -1$

$x_2 = \frac{1}{2}$

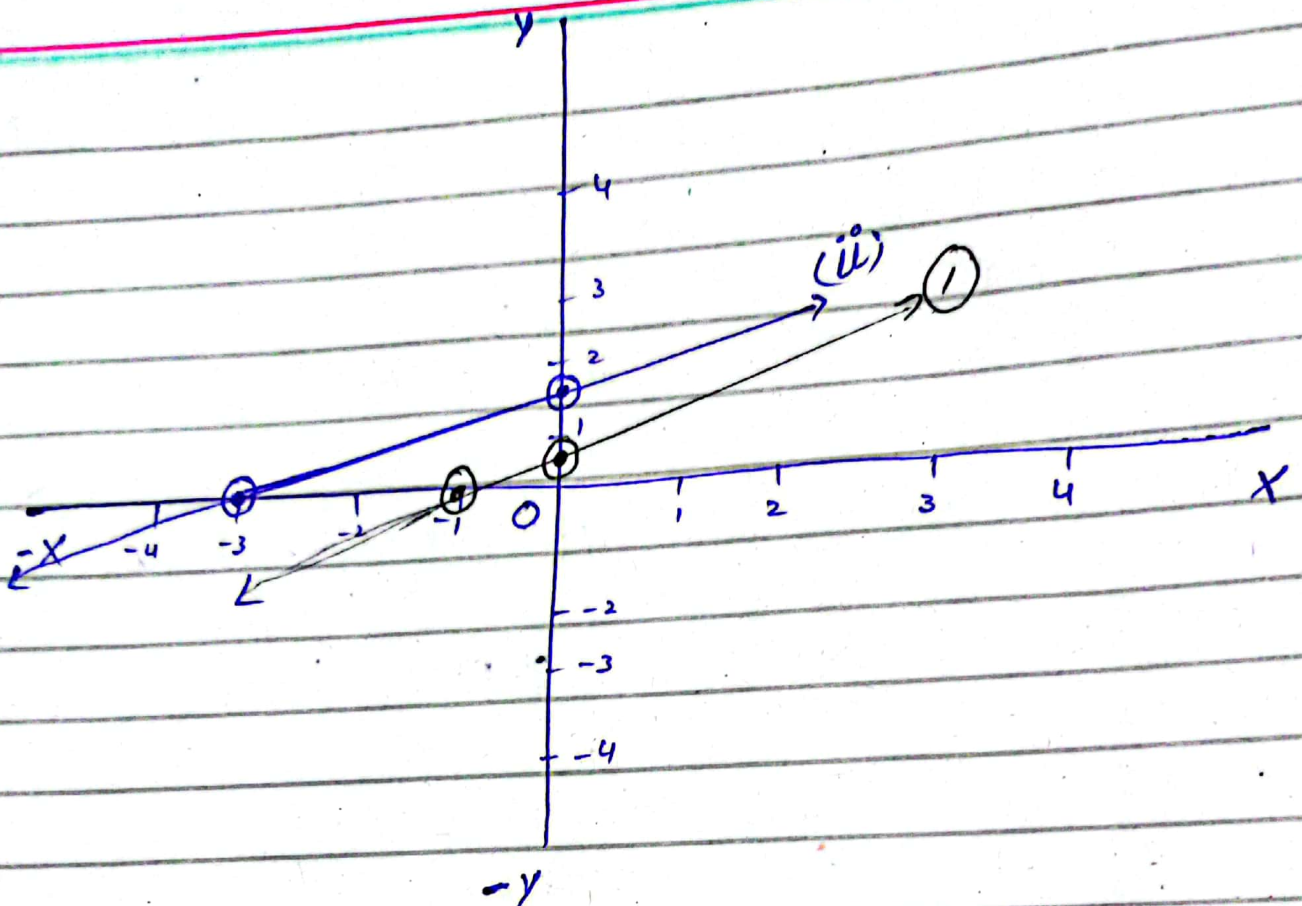
$x_2 = 0.5$

A (0, 0.5)

Put $x_2 = 0$ in ①

$x_1 = -1$

B(-1, 0)



Put $x_1 = 0$ in (ii)

$x_2 = \frac{3}{2}$

$x_2 = 1.5$

$\therefore$ $C = (0, 1.05)$

put $x_2 = 0$ in (ii)

$x_1 = -3$

$D = (-3, 0)$

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(ii) A system of linear equation has exactly one solution.

$$x_1 - 2x_2 = -1 \quad \text{——} \ ①$$
$$-x_1 + 3x_2 = 3 \quad \text{——} \ ②$$

Put $x_1 = 0$ — ①

$-2x_2 = -1$

$x_2 = \frac{1}{2}$

$x_2 = 0.5$

$A(0, 0.5)$

put $x_2 = 0$ in ①

$x_1 = -1$

$B(-1, 0)$

put $x_1 = 0$ in (ii)

$x_2 = 1$

$C(0, 1)$

put $x_2 = 0$ in (ii)

$x_1 = -3$

$D = (-3, 0)$

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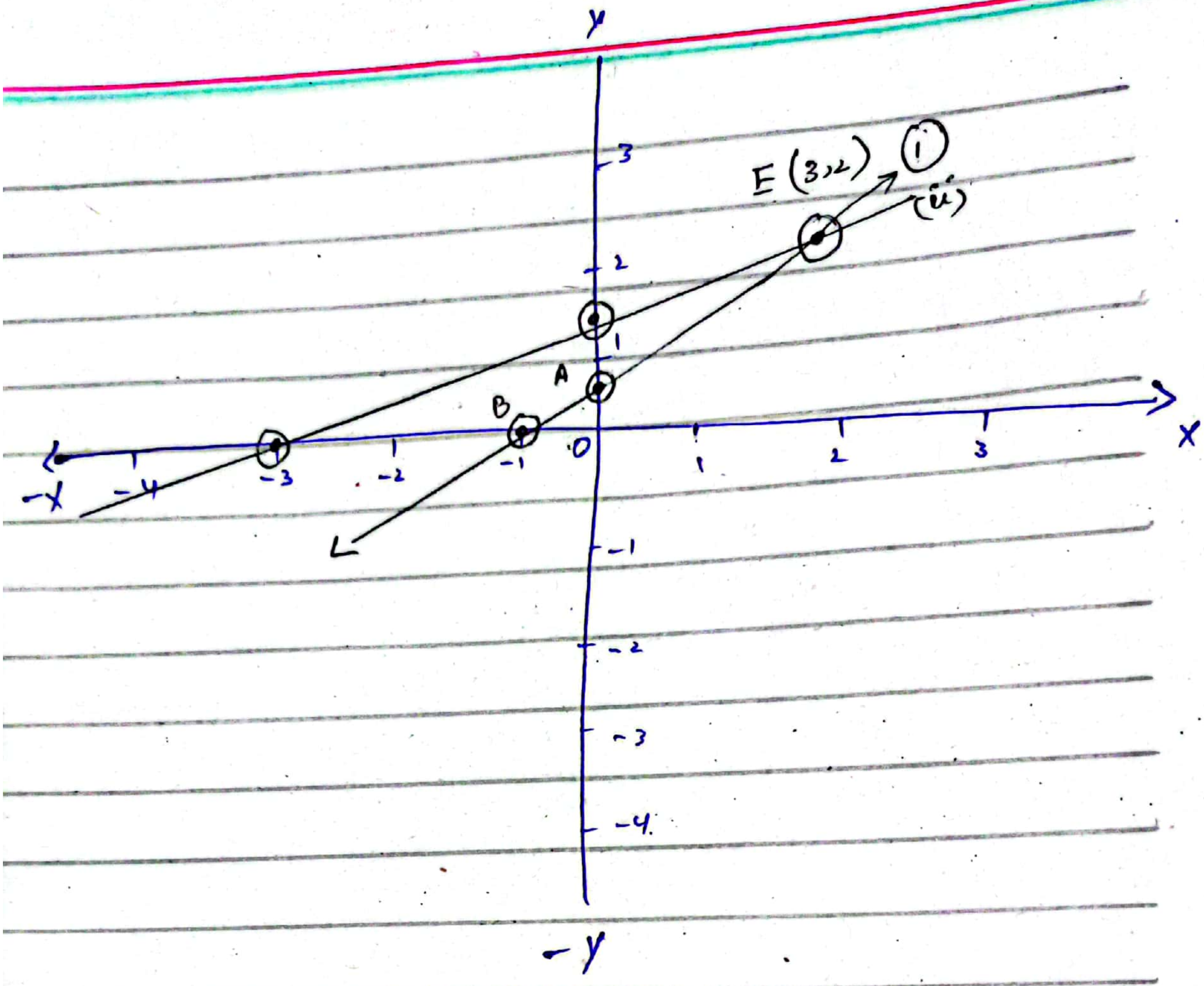


eq adding (i) & (ii)

$$\boxed{x_2 = 2}$$

Put in (i)

$$x_1 - 2(2) = -1$$

$$x_1 - 4 = -1$$

$$x_1 = 4 - 1$$

$$\boxed{x_1 = 3}$$

(iii) A system of linear equation has infinitly many solutions.

$x_1 - 2x_2 = -1$ ——— (i)

$-x_1 + 2x_2 = 1$ ——— (ii)

Put $x_1 = 0$ in (i)

$-2x_2 = -1$

$x_2 = \frac{1}{2}$

$x_2 = 0.5$

A (0, 0.5)

put $x_2 = 0$ in (i)

$x_1 = -1$.

B (-1, 0)

=> B (0, -1)

Put ($x_1 = 0$) in (ii)

$x_2 = \frac{1}{2}$

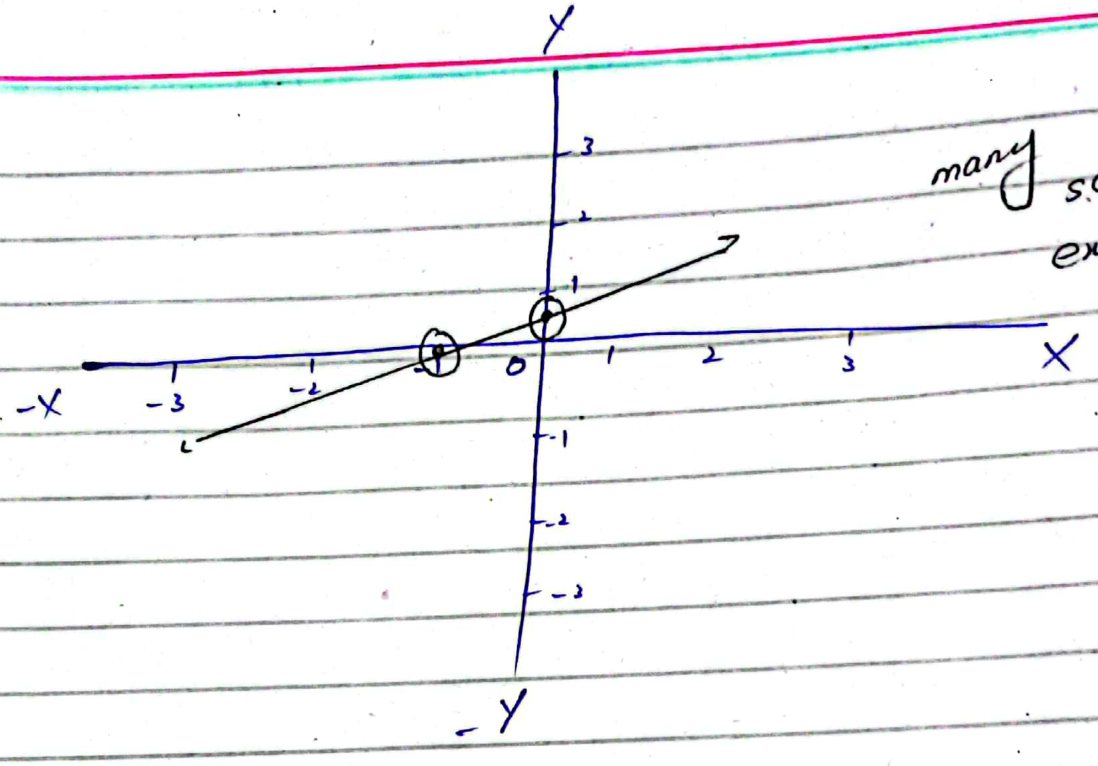
C (0, 0.5)

Put $x_2 = 0$ in (ii)

$x_1 = -1$

D = (-1, 0)

D (0, -1)



many solution exist

Matrix notation of linear system

(2)
$$2x_1 - x_2 + x_3 = 0$$
$$0x_1 + 2x_2 - 8x_3 = 8$$
$$-4x_1 + 5x_2 + 9x_3 = -9$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 8 \\ -9 \end{bmatrix}$$

$$AX = B$$

which is matrix form of linear system.

Augmented matrix

$$A_b = \left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right]$$

* Elementary Rows operations.

1) Add a constant times a row (equation) to another.

2) interchange any two rows (equations)

3) Multiply a row (equation) by a non zero constant.

* leading Entry of a row

leftmost non-zero entry in a non-zero row

$$\begin{bmatrix} 2 & -3 & 2 & 5 \\ 0 & 1 & -4 & 6 \\ 0 & 0 & 0 & 7/2 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 6 & 5 \end{bmatrix}, \begin{bmatrix} 0 & -3 & 5 & 9 \\ -1 & 2 & 0 & 5 \\ -2 & -3 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Echelon form of a matrix

A matrix is in echelon form if it has following three properties.

- i) All non-zero rows are above any rows of all zeros.

ii) Each leading entry of a row is in a column to the right of the leading entry of the row.

iii) All entities in a column below a leading entry are zero.

Exp:- leading entry

$$\begin{bmatrix} 1 & 4 & -3 & 7 \\ 0 & 1 & 6 & 5 \\ 0 & 0 & 2 & 6 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

imp Point

- 0 rows نیچے ہوں اور non-zero اوپر
- row کی leading entry right پہ ہو۔
- leading entry کے نیچے 0 ہو ہمیشہ۔

Exp :-

(i) $\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ II, III not satisfied

(ii) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & 1 & 0 \end{bmatrix}$ II, II not satisfied

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ I, II, III condition are not satisfied

So these are not echelon form.

Exp :- convert echelon form

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{bmatrix}$$ By $R_2 + (-3)R_1$

$-3R_1 \leftarrow$	-3	-9	-12	-21
$R_2 \leftarrow$	3	9	7	6
	0	0	-5	-15

Convert Echelon form

Exp :-

$$\begin{bmatrix} 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \qquad R_{12}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix} \qquad \begin{array}{l} R_2 + (-4)R_1 \\ R_3 + (-6)R_1 \end{array}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix} \qquad \begin{array}{l} \left(\frac{1}{-3}\right)R_2 \\ \left(-\frac{1}{5}\right)R_3 \end{array}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_3 + (-1)R_2$$

Which is echelon form.

Home Task :-

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & 2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

Reduce Echelon form of a matrix

5 Properties

- i) All non zero rows are above any rows of all zeros.
- ii) Each leading entry of a row is in a column to the right of the leading entry of the row above it.
- iii) All entries in a column below a leading entry are zeros.
- iv) The leading entry in each non-zero row is 1.
- v) Each leading 1 is the only non-zero entry in its column.

Exp

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

→ reduce echelon form.

Exp :-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$ I , III , IV V

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ IV

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ II , III , V

not satisified

Exp convert reduce echelon form.

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & 4 \end{bmatrix} \quad R_2 + (-2)R_1$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix} \quad (-1)R_2$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 4 \end{bmatrix} \qquad R_1 + (-4)R_2$$

which is reduce. echelon form

Exp:-

$$\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 3 & -4 & 2 & 0 \\ -3 & -4 & 2 & 0 \\ 3 & -4 & 2 & 0 \end{bmatrix} \qquad \begin{matrix} \left(-\frac{1}{3}\right)R_2 \\ \left(-\frac{1}{2}\right)R_3 \end{matrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & -4/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \frac{1}{3}R_1$$

which is reduce. echelon form.

(i) Guass Jordan Elimination method:-

→ Reduce Echelon form

Open nichy zero

(ii) Guassian Elimination method:-

→ Echelon form.

1)

$$\overset{R}{\sim}\left[\begin{array}{cc|c} & & \\ & & \\ & 0 & -6 \end{array}\right]$$ no solution.

2)

$$\overset{R}{\sim}\left[\begin{array}{cccc|c} & & & & \\ & & & & \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$ Arbitrary solution.

3)

$$\overset{R}{\sim}\left[\begin{array}{cc|c} & & \\ & 4 & -6 \end{array}\right]$$ unique solution.

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Rank of matrix

→ no of non-zeros rows in echelon form of a matrix.

Inconsistent solution (no solution)
→ solution not exist.

Rank of A $\neq$ Rank of A_b
<

$$R \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

* non zeros rows is called rank.

Rank of A = 2.

Rank of A_b = 3

Consistent system
→ solution exist

unique. (trivial)

Arbitary (non-trivial)

Rank of A = Rank of A_b

* in arbitrary :-

Rank A = Rank A_b < no of variables.

in unique :-

Rank of A = Rank of A_b = no of varieb

20/July/2023

→ Guass - Jordon Elimination method (reduce echelon form)

→ Gauss (Gaussian) Elimination method Echelon form.

Gauss Jordon method :-

Exp 1 :-

$$2x_1 - x_2 - x_3 = 4$$
$$3x_1 + 4x_2 - 2x_3 = 11$$
$$3x_1 - 2x_2 + 4x_3 = 11$$

$$A_b = \begin{bmatrix} 2 & -1 & -1 & 4 \\ 3 & 4 & -2 & 11 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$R_2 - R_1$

$$\underset{\sim}{R} = \begin{bmatrix} 2 & -1 & -1 & 4 \\ 1 & 5 & -1 & 7 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

R_{12}

$$\underset{\sim}{R} \begin{bmatrix} 1 & 5 & -1 & 7 \\ 2 & -1 & -1 & 4 \\ 3 & -2 & 4 & 11 \end{bmatrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 5 & -1 & 7 \\ 0 & -11 & 1 & -10 \\ 0 & -17 & 7 & -10 \end{bmatrix} \qquad \begin{matrix} R_2 + (-9)R_1 \\ R_3 + (-3)R_1 \end{matrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 5 & -1 & 7 \\ 0 & 1 & -\frac{1}{11} & \frac{10}{11} \\ 0 & -17 & 7 & -10 \end{bmatrix} \qquad \left(-\frac{1}{11}\right)R_2$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 0 & \frac{-6}{11} & 27/11 \\ 0 & 1 & -1/11 & 10/11 \\ 0 & 0 & \frac{60}{11} & \frac{60}{11} \end{bmatrix} \qquad \begin{matrix} R_1 + (-5)R_2 \\ R_3 + 17 R_2 \end{matrix}$$

$$\overset{R}{\sim}\begin{bmatrix} 1 & 0 & \frac{-6}{11} & \frac{27}{11} \\ 0 & 1 & -1/11 & 10/11 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \frac{11}{60}R_3$$

$$\overset{R}{\sim}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad \text{by} \quad \begin{matrix} R_2 + \frac{1}{11}R_3 \\ R_1 + \frac{6}{11}R_3 \end{matrix}$$

$x_1 = 3$

$x_2 = 1$

$x_3 = 1$

S.set$(3, 1, 1)$

Exp # 2

$$0x_2$$

$$5x_1 + 4x_3 + 2x_4 = 3$$

$$x_1 - x_2 + 2x_3 + x_4 = 1$$

$$4x_1 + x_2 + 2x_3 + 0x_4 = 1$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$Ab = \begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ interchange R_{12}

$$\overset{R}{\sim} \begin{bmatrix} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{bmatrix}$$ $R_2 + R_1(-5)$, $R_3 + R_1(-4)$, $R_4 + R_1(-1)$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & -6/5 & -3/2 & -2/3 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{bmatrix}$$ $R_2\left(\frac{1}{5}\right)$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 7/5 & 6/5 & -1/5 \end{bmatrix} \quad \begin{array}{l} R_1 + R_2(1) \\ R_3 + R_2(-5) \\ R_4 + R_2(-2) \end{array}$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 7/5 & 6/5 & -1/5 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \quad R_{34}$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/7 \\ 0 & 0 & 1 & 6/7 & -1/7 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \quad \frac{5}{7} R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-2}{7} & \frac{5}{7} \\ 0 & 1 & 0 & 3/7 & -16/35 \\ 0 & 0 & 1 & \frac{6}{7} & -1/7 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \quad \begin{array}{l} R_1 + \left(\frac{-4}{5}\right) R_3 \\ R_2 + \left(6/5\right) R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2/7 & 5/7 \\ 0 & 1 & 0 & 3/7 & -16/35 \\ 0 & 0 & 1 & 6/7 & -1/7 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad R_4(-1)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 6/7 & -1/7 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} R_1 + \left(\frac{2}{7}\right) R_4 \\ R_2 + \left(\frac{-3}{7}\right) R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 + \left(-\frac{6}{7}\right) R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$x_1 = 1$, $x_2 = -1$, $x_3 = -1$, $x_4 = 1$

Solution Set $(1, -1, -1, 1)$ A.

$$A \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 12 \\ 0 & 0 & 0 & 1 & 20 \end{array}\right]$$

Exp # 3 :-

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1.5 & 68 \\ 3 & 4 & 2.5 & 2.5 & 142 \\ 1 & 2 & 1.5 & 1 & 64 \\ 10 & 10 & 9 & 8 & 448 \end{array}\right]$$

Gauss (Gaussian Elimination method)

Exp:-

$$x_1 - x_2 + 2x_3 = 0$$
$$4x_1 + x_2 + 2x_3 = 1$$
$$x_1 + x_2 + x_3 = -1$$

Sol:-

$$Ab = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

1 Leading entry nichy 0,0 bnana

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 5 & -6 & 1 \\ 0 & 2 & -1 & -1 \end{bmatrix} \quad \begin{matrix} R_2 + (-4)R_1 \\ R_3 + (-1)R_1 \end{matrix}$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -4 & 3 \\ 0 & 2 & -1 & -1 \end{bmatrix} \quad R_2 + (-2)R_3$$

$$\overset{R}{\sim} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 7 & -7 \end{bmatrix} \quad R_3 + (-2)R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \frac{1}{7}R_3$$

by backward substitution.

$$x_1 - x_2 + 2x_3 = 0 \quad \text{—— (1)}$$
$$x_2 - 4x_3 = 3 \quad \text{—— (2)}$$
$$\boxed{x_3 = -1} \quad \text{—— (3)}$$

Put in eq (2)

$$x_2 - 4(1) = 3$$
$$x_2 = 4 - 3$$
$$x_2 = 3 - 4$$
$$\boxed{x_2 = -1}$$

put in eq (1)

$$x_1 - (-1) + 2(-1) = 0$$
$$x_1 + 1 - 2 = 0$$

$x_1 - 1 = 0$

$\boxed{x_1 = 1}$

Exp :-

$x_1 + 2x_2 + x_3 = 160$

$2x_1 + x_2 + x_3 = 2000$

$2x_1 + 2x_3 = 240$

Ans

$(60, 20, 60)$

Exp 2 :-

$$\begin{bmatrix} 1 & 5 & 2 & 9 \\ 1 & 1 & 7 & 6 \\ 0 & -3 & 4 & -2 \end{bmatrix}$$

Ans

$(-3, 2, 1)$

Pivot Position of a matrix

A location in a matrix A corresponds to a leading entry in row echelon form.

$$\begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

free variables

$\downarrow \quad \downarrow \quad \downarrow$

pivot coloumn

(جس میں leading entry ہے)

Basic Variable :-
A variable that corresponds to a pivot column is said to be basic variable.

free Variable :-
A variabe that corresponds to a

non- pivot column is said to be free variable.

→ x_1, x_2, x_4 are basic variables.

→ x_3. are free variable.

—.

"General solution of a linear system"

Exp 1:-

$$Ab = \left[\begin{array}{ccc|c} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{array}\right]$$

$$\underset{R}{\sim} \left[\begin{array}{ccc|c} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{array}\right] \quad R_2 + (-2)R_1$$

$\downarrow \quad \downarrow$

Pivot column

x_1, x_2 are basic varible.

x_3 is free variable.

So, given abover system has general Sol because x_3 is free.

by backward substituation.

$x_1 + 4x_2 = 7$ — (1)

$-x_2 = -4$ — (2) $\Rightarrow x_2 = 4$.

$x_3 = $ free.

Put in eq (1)

$x_1 + 4(4) = 7$

$x_1 + 16 - 7$

$x_1 = 7 - 16$

$\boxed{x_1 = -9}$

Thus,

$\Rightarrow x_1 = -9, \; x_2 = 4, \; x_3 = t$

so, t is arbitrary.

Exp #02

$$A_b = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \left[\begin{array}{ccccc|c} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \\ \checkmark & \checkmark & & \checkmark & & \end{array}$$

pivot columns.

x_1, x_2, x_5 are basic variables.

x_3, x_4 are free variable.

so the given system has general sol. because x_3 and x_4 are free variable

$$x_1 + 2x_2 - 5x_3 - 6x_4 = -5 \quad \text{— (1)}$$

$$x_2 - 6x_3 - 3x_4 = 2 \quad \text{— (2)}$$

$$x_5 = 0$$

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let $x_3 = t$, $x_4 = s$

from eq (ii)

$$x_2 - 6t - 3s = 2$$

$$\boxed{x_2 = 2 + 6t + 3s}$$

from (i)

$$x_1 + 2(2 + 6t + 3s) - 5t - 6s = -5$$
$$x_1 + 4 + 12t + 6s - 5t - 6s = -5$$
$$x_1 + 7t = -5 - 4$$

$$\boxed{x_1 = -9 - 7t}$$

Test the consistency of linear system

Ex :-

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = -13$$
$$2x + 19y - 47z = 22$$

$$Ab = \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 22 \end{array}\right]$$

Rank of Ab = 3

Rank of A = 2

Since Rank of Ab ≠ Rank of A

∴ the system is inconsistent

& exist no solution

system [illegible] Rank

consistent

Exp #02

$$x + y + z = 6$$
$$2x + y + 3z = 13$$
$$5x + 2y + z = 12$$

$$Ab = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 13 \\ 5 & 2 & 1 & 12 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -7 & 21 \end{array}\right]$$

solution
b/c unique 3 non leading entry

Rank of Ab = 3
Rank of A = 3
no of variable = 3

Since rank of Ab = no of variable.

∴ the given system is consistent and exist unique solution.

By back work subsitutions.

$x + y + z = 6$ — (i)

$-y + z = 1$ — (ii)

$-7z = -21$ — (iii)

from (iii) $z = 3$

from (ii) $y = 2$

from (i) $x = 1$