PAST

Past Paper 2023

Short Questions.

1. Explain the different parts of Proposition function with an example.

The different parts of propositional functions are follow as:

- **Variables:** P, Q and r are variables
- **Constants:** "If then ($\rightarrow$)" and "and ($\wedge$)".
- **Logical Connectives:** ($\rightarrow, \wedge, \vee, \leftrightarrow, \oplus$)
- **Proposition:** Which has a fixed truth value.
- **Truth Value:** The truth value of propositional logic depends on the truth value of its variables.

Day: Date:

2. How many edges does a graph have if its degree sequence is 5, 2, 2, 2, 2, 1? Draw such a graph.

Sum of degree = 5+2+2+2+2+1

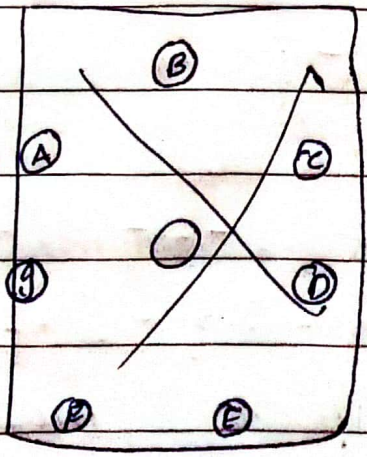
= 14

Total edges = $\frac{\text{Sum of degree}}{2}$

$E = \frac{14}{2}$

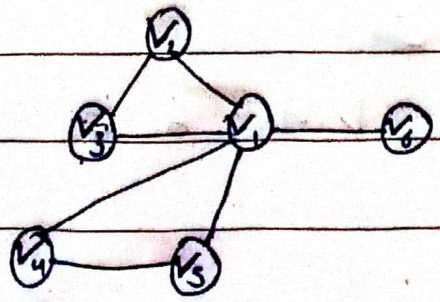
E = 7 edges

Graph



Adjacency list

- V_1 : V_2, V_3, V_4, V_5, V_6
- V_2 : V_1, V_3
- V_3 : V_1, V_2
- V_4 : V_1, V_5
- V_5 : V_1, V_4
- V_6 : V_1



Day: Date:

3. Determine wether the sentence "To enter the country you need a password or a voter registration card." is an exclusive -OR, or Inclusive-OR or intended. explain your answer.

Solve:-

This sentence is an example of an inclusive-OR. Because in an inclusive-OR it implies that you can use either a password or a registration card. The sentence suggests that having either one or both.

4. Encrpt the cipher message "UNIVERSITY OF THE PUNJAB" using the shift cipher with shif $k=7$.

The encrypted message is:

"BUPCLYZPAE VM AOL WBUQHI"

5. What rule of inference are used in the given argument?

Day: Date:

"If I will study discrete math, then I will study computer science. Therefore, if I will study discrete math, then I will study discrete mathematics and I will stud computer science."

Solve:-

The rules of inference are used in argument are:

- **Transitivity:** This rule allows to "chain together" $P \to q$, $q \to r$, $P \to r$
- **Addition:** This rule allows to "combine" $P, q \to P \wedge q$
- **Simplification:** This rule allows to "break apart" $P \wedge q \to P$

6. Use a direct proof to show that the sum of two even integers is even.

Proof

Let two even integers

$x = 2a$ — ①

$y = 2b$ — ② (where a, b are i

Add ① and ②

$x + y = 2a + 2b$

$x + y = 2(a + b)$

7.5
+ 7.5
14.0
- 7.5
7.5

Day Date

Since $a+b$ is an integers.

we can see that $2(a+b)$ is an even integer.

8. Evaluate the following summation using $x=2$, $y=3$, $b=2.5$ and $c=3$.

$$\frac{\sum_{j=0}^{y}\sum_{i=1}^{c} j^2 + i - [bc]}{\sum_{i=0}^{x} 2 + i^2}$$

Solve

$$= \sum_{j=0}^{3}\sum_{i=1}^{3} j^2 + i - (3)(2.5) \quad \text{inner summation}$$

$$= \sum_{j=0}^{3}\sum_{i=1}^{3} j^2 + i - 7.5$$

expand

$$= \sum_{j=0}^{3}\left[(j^2 + 1 - 7.5) + (j^2 + 2 - 7.5) + (j^2 + 3 - 7.5)\right]$$

$$= \sum_{j=0}^{3}(j^2 + 1 - 7.5 + j^2 + 2 - 7.5 + j^2 + 3 - 7.5$$

outer summation

$$= \sum_{j=0}^{3} 3j^2 - 16.5$$

expand

$$= (3(0)^2 - 16)^{.5} + (3(1)^2 - 16)^{.5} + (3(2)^2 - 16)^{.5} + (3(3)^2 - 16)^{.5}$$

Day: Date:

~~$E = -16.5 - 13 = 6 + 9 = 30$~~ $= -16.5 - 13.5 - 4.5 + 10.5$

$= -24$

$$\sum_{i=0}^{x} 2 + i^2 = \sum_{i=0}^{2} 2 + i^2$$

expand

$= (2 + 0^2) + (2 + 1^2) + (2 + 2^2) = 2 + 3 + 6$

$= 11$

x by x $\quad \because x = 2$

$= 11 \times 2 = 22$

$= \frac{-24}{22} = \frac{-12}{11}$

$= -1.09$ Ans

7. What will be next term in the sequence 7, 11, 19, 35, ... ?

Let

diff b/w consecutive terms

$11 - 7 = 4$

$19 - 11 = 8$

$35 - 19 = 16$

The diff b/w

$= (4, 8, 16)$ which are power of 2

$= (2^2, 2^3, 2^4)$

following this pattern

$2^5 = 32$

Next term

$= 35 + 32$

$= 67$ Ans

9. Let A be the set $\{2, 4, 6, 8, 9\}$. In $A \times A$ which order pair in relation $R = \{(x, y) \mid x \geq y + 1\}$. Determine wether the relation R on the set A is reflexive, symmetric or transitive.

Solve:

$R = \{(4,2), (6,2), (6,4), (8,2), (8,4), (8,6), (9,2), (9,4), (9,6), (9,8)\}$

R is not reflexive

R is not symmetric

Transitive

(4, 2)

(6, 2)

(6, 4) (6, 2)

(8, 2)

(8, 4) (8, 2)

(8, 6) (8, 4)

(9, 2)

(9, 4) (9, 2)

(9, 6) (9, 4)

(9, 8) (9, 6)

Day: Date:

R is Transitive

10. Prove that the wether conditioned statements $(P \to Q) \wedge (Q \to R) \to (P \to R)$ is a tautology or not using laws of equivalence.

Solve:-

$(P \to Q) \wedge (Q \to R) \to (P \to R)$

$(\sim P \vee Q) \wedge (\sim Q \vee R) \to (P \to R)$ [using equivalence $P \to Q \equiv \sim P \vee Q$, $Q \to R \equiv \sim Q \vee R$]

$(\sim P \vee Q) \wedge (\sim Q \vee R) \to (\sim P \vee R)$ $\because P \to R \equiv \sim P \vee R$

$(\sim P \vee Q) \wedge (\sim Q \vee R) \equiv (\sim P \wedge \sim Q) \vee (\sim P \wedge R) \vee (Q \wedge \sim Q) \vee (Q \wedge R)$ [use the distributive Law]

$\equiv (\sim P \wedge \sim Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

$(\sim P \wedge \sim Q) \vee (\sim P \wedge R) \vee (Q \wedge R) \to (\sim P \vee R)$ [use the absorption Law]

$\equiv (\sim P \vee R)$

The statement is a tautology.

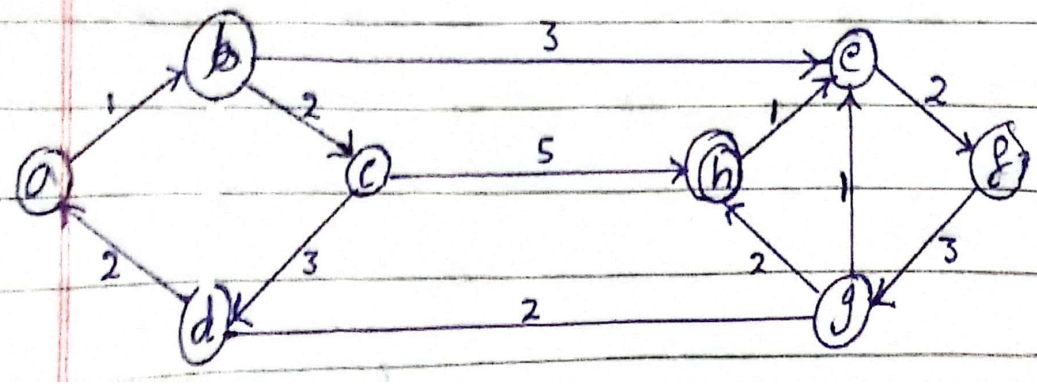
11. How many different elements does A^n have when A has m elements and n is a positive integer?

- A^n, also denoted as $A \times A \times \ldots \times A^n$, represents the Cartesian product of set A with itself n times.
- A has m element.
- $m \times m \times \ldots \times m^n$

So, A^n has m^n different elements.

Long Questions

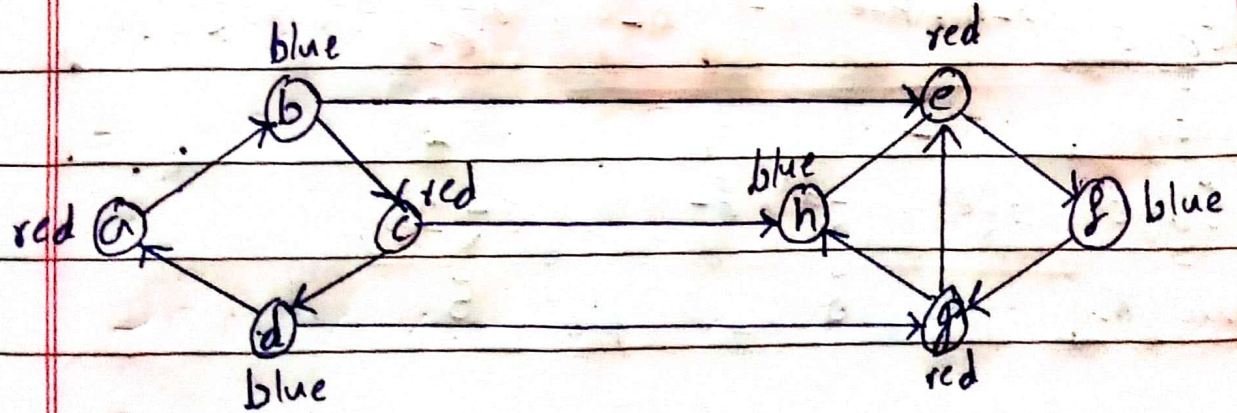
2. Determine the chromatic number, calculate the degree of each vertex and incidence matrix of a given Graph. Furthermore, find the minimum spanning tree using Kruskal's algorithm.



Degree

In-degree	out-degree	Total-degree
$\deg^-(a) = 1$	$\deg^+(a) = 1$	$\deg(a) = 2$
$\deg^-(b) = 1$	$\deg^+(b) = 2$	$\deg(b) = 3$
$\deg^-(c) = 1$	$\deg^+(c) = 2$	$\deg(c) = 3$
$\deg^-(d) = 1$	$\deg^+(d) = 2$	$\deg(d) = 3$
$\deg^-(e) = 3$	$\deg^+(e) = 1$	$\deg(e) = 4$
$\deg(f) = 1$	$\deg^+(f) = 1$	$\deg(f) = 2$
$\deg(g) = 2$	$\deg^+(g) = 3$	$\deg(g) = 5$
$\deg^-(h) = 2$	$\deg^+(h) = 1$	$\deg(h) = 3$

Chromatic Number:-



$\boxed{\chi C_8 = 2}$ Ans

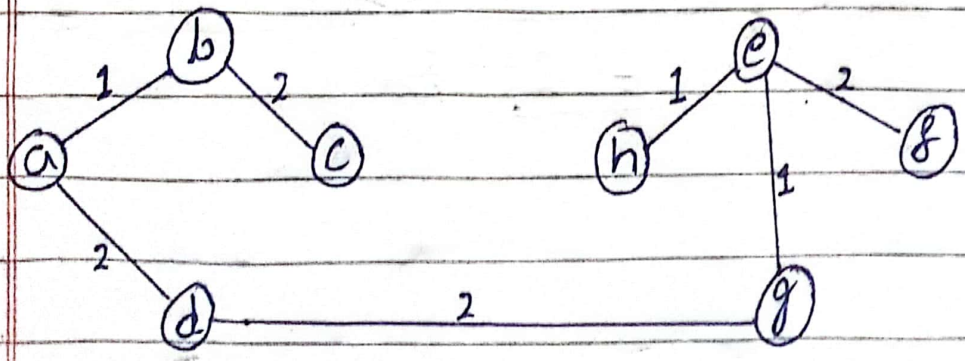
Incidence Matrix:-

	1	2	3	5
a	1	-1	0	0
b	-1	1	1	0
c	0	-1	1	1
d	0	1	-1	0
e	-1	1	-1	0
f	0	-1	1	0
g	1	1	-1	0
h	1	-1	0	-1

Kruskal's Algorithm:-

Edge	(a-b), (h-e), (g-e)	(b-c), (d-a), (d-g), (g-h), (e-f)
weight	1	2

Edge	(b-e), (c-d), (f-g)	(c-h)
weight	3	5



Proved that

1. Use mathematical induction to prove that for any positive integer number n.

$$2^2+5^2+8^2+\cdots+(3n-1)^2=\frac{n(6n^2+3n-1)}{2} \quad \text{---(1)}$$

Solve:-

For $n = 1$

$$2^2 = \frac{1(6(1)^2+3(1)-1)}{2}$$

$$4 = \frac{8}{2}$$

$$4 = 4$$

This show that (1) is true for $n=1$.

Step 2:-

Let (1) be true for

$n = k \in N$.

$$2^2+5^2+8^2+\cdots+(3k-1)^2 = \frac{k(6k^2+3k-1)}{2} \quad \text{---(2)}$$

For $n = k+1$ eq (1)

$$2^2+5^2+8^2+\cdots+(3(k+1)-1)^2 = \frac{(k+1)[6(k+1)^2+3(k+1)-1]}{2}$$

$$2^2+5^2+8^2+\cdots+(3k+2)^2 = \frac{(k+1)(6k^2+12k+6+3k+3-1)}{2}$$

Day: Date:

$$2^2+5^2+8^2+\cdots+(3k+2)^2=\frac{(k+1)(6k^2+15k+8)}{2}$$

Adding $(3k+2)^2$ on b/s eqv (2)

$$2^2+5^2+8^2+\cdots+(3k-1)^2+(3k+2)^2=\frac{k(6k^2+3k-1)}{2}+(3k+2)^2$$

R.H.S

$$=\frac{k(6k^2+3k-1)+2(3k+2)^2}{2}$$

$$=\frac{6k^3+3k^2-k+18k^2+24k+8}{2}$$

$$=\frac{6k^3+21k^2+23k+8}{2}$$

$$=\frac{(k+1)(6k^2+15k+8)}{2}$$

$$=\frac{(k+1)(6k^2+12k+6+3+3-1)}{2}$$

$$=\frac{(k+1)[6(k+1)^2+3(k+1)-1]}{2}$$ AZS

Proved that

نوٹ:

جہاں ہم نے eqv (1) میں Add $(k+1)$ کیا ہے اور اس کو solve کیا۔ اور اس کے بعد ہم نے eqv (2) میں $(3k+2)^2$ Add b/s کر دیا پھر R.H.S کی solve کی اور اس کے

بعد اگر آپ غور کرے کے ہم نے جب eq ① میں Add (k+1) کرکے eq کی solve کی تھی جہاں تک اس کو solve کیا پھر ہمارا conversely ⇐ ہو گیا۔

3. Using counting and probability concepts, answer the following:

(a) A sequence of 10 bits is randomly generated. What is the probability that at least one of these bit is 0?

Solve

There are the possible sequence

$$2^{10} = 1024$$

one bit is zero so,

$$1024 - 1 = 1023$$

Probability = (Number of sequence) / Total number of sequence

$$= \frac{1023}{1024}$$

$$= 0.99 \text{ Ans}$$

c) How many possibilities are there for the win, place, and show

Day: Date:

position in a horse race with 20 horses if all orders of finish are possible?

Solve:-

$r = 3$

No of horses $= 20$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$${}^{20}P_{3} = \frac{20!}{(20-3)!}$$

$$= \frac{20.19.18.\cancel{17!}}{\cancel{17!}}$$

$$= 6,840 \text{ Ans}$$

(d) A group contains n men and m women. How many ways are there to arrange these people in a row if the men and women alternate?

Solve

men $= n!$

women $= n!$

men × women × 2

$2 \times n! \times n!$

$2(n!)^2$ Ans

(e) How many bit string of length ten both begin and end with a 1?

Solve

The bit strings are

$= 1.2.2.2.2.2.2.2.2.1$

$= 2^8$

$= 256$ Ans

There are 256 bit strings are in length ten.

Day: _____ Date: _____

(b) What is the co-efficient of x^9 in $(2-x)^{19}$?

Solve

Binomial Theorem states that

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$a = 2$, $b = -x$ and $n = 19$

$k = 9$, $x = 1$

$$= \binom{19}{9} 2^{19-9} (-1)^9$$

$${}^{19}C_9 = 92,378$$

$$2^{19-9} = 1024$$

$$(-1)^9 = -1$$

$$= 92,378 \times 1024 \times (-1)$$

$$= -94,595,072 \text{ Ans}$$