Math
Past Paper
Fall 2023

Short Questions

Q#1: Can you use the principal of mathematical induction to determine whether a given formula for the sum of the first n terms of sequence are correct?

Sol:

Yes, the principal of mathematical induction can be used to verify the correctness of a formula for the sum of first n terms of sequence.

→ Verify the formula is true for $n=1$

→ Assume the formula is true for $n=K$

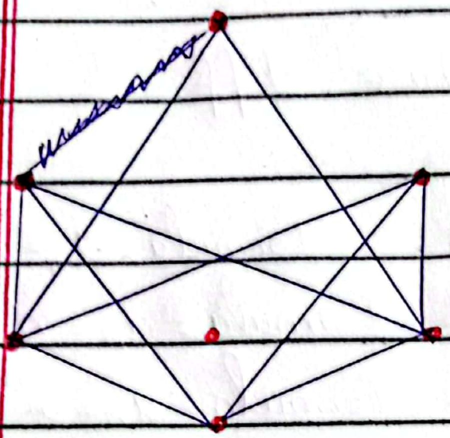
→ Prove that if the formula is true for $n=K$ then it will also true for $n=K+1$.

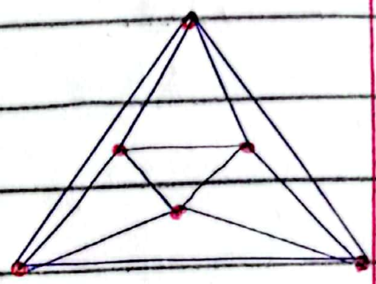
→ Conclude the formula is true for all Positive integers n.

Q#2: Draw graphs:

(i) K123 (ii) K222 (iii) K1223

Sol.





K222

octahedral graph

K123

Fan graph

Q#3: How many ways are there to put in identical objects into **m** distinct containers so that no container is empty?

Sol. Now by combination:

$$^{n-1}C_{m-1} = \frac{(n-1)!}{(n-1-m+1)!\,(m-1)!}$$

$$= \frac{(n-1)!}{(n-m)!\,(m-1)!}$$

Q#4: Find at least three different sequences begining with the terms **1, 2, 4** whose terms are generated by a simple formula or rule?

Sol: Here are three different sequences:

(i) 1, 2, 4, 7, 11 ----- Formula = $a_n = n^2 - n + 2$

(ii) 1, 2, 4, 9, 16 ---- Formula = $a_n = n^2 + 1$

(iii) 1, 2, 4, 8, 16 ----- Formula = $a_n = 2^{n-1}$

Q#5: Let **a** and **b** be real numbers with **a<b**. Use the floor and ceiling function to express the number of integers **n** to satisfy inequality **a<n<b**.

Sol:

$$\lfloor b \rfloor + \lceil a \rceil + 1$$

→ $\lceil a \rceil$ is the smallest integer greater than or equal to **a** (ceiling function)

→ $\lfloor b \rfloor$ is greater integer less than or equal to **b** (floor function)

→ Subtracting $\lceil a \rceil$ and $\lfloor b \rfloor$ give number of integers between a and b

→ Adding 1 includes the itger b but not a, So a is not integer satisfying inequality

LONG Questions

Q#1. Prove that the associative law of boolean algebra can be omitted from defination.

Sol

Commutative laws

$x \wedge y = y \wedge x$ (AND operation is Comm)

$x \vee y = y \vee x$ (OR operation is Comm)

Associative laws

$x \wedge (y \wedge z) = (x \wedge y) \wedge z$ (AND Operation is Ass)

$x \vee (y \vee z) = (x \vee y) \vee z$ (OR Operation is Ass)

Distributive laws

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ (AND Dis over OR)

$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ (OR Dis over AND)

Identity laws

$x \wedge 1 = x$ (AND with 1 is identity)

$x \vee 0 = x$ (OR with 0 is identity)

Complement laws

$x \wedge \sim x = 0$ (AND with complement 0)

$x \vee \sim x = 1$ (OR with complement 1)

Absorbtion laws

$x \wedge (x \vee y) = x$ (AND absorbs OR)

$x \vee (x \wedge y) = x$ (OR absorbs AND)

Demorgan laws

$\sim(x \wedge y) = \sim x \vee \sim y$ (NOT distributes over AND)

$\sim(x \vee y) = \sim x \wedge \sim y$ (NOT distributes over OR)

Q#2. Describe a sample space and events **A, B** and **C** where **P(A\cap B\cap C) = P(A)\cdot P(B) \cdot P(C) but A, B** and **C** not pairwise independent.

Sol Lets Consider

Sample space $= S = \{1, 2, 3, 4, 5, 6\}$

Events

$A = \{1, 2, 3\}$ (getting roll of 1, 2 and 3

$B = \{1, 3, 5\}$ (getting roll of 1, 3 and 5

$C = \{2, 3, 4\}$ (getting roll of 2, 3 and 4

Probability

$P(A) = 3/6 = \frac{1}{2}$

$P(B) = 3/6 = \frac{1}{2}$

$P(C) = 3/6 = \frac{1}{2}$

Intersection of Events

$A \cap B = \{1, 2, 3\} \cap \{1, 3, 5\} = \{1, 3\}$

$A \cap C = \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$

$B \cap C = \{1, 3, 5\} \cap \{2, 3, 4\} = \{3\}$

$A \cap B \cap C = \{1, 2, 3\} \cap \{1, 3, 5\} \cap \{2, 3, 4\} = \{3\}$

Probability of intersection

$P(A \cap B) = 2/6 = \frac{1}{3}$

$P(A \cap C) = 2/6 = \frac{1}{3}$

$P(B \cap C) = 1/6$

$P(A \cap B \cap C) = 1/6$

Since $P(A \cap B \cap C) \neq P(A) \times P(B) \times P(C)$ events A, B and C are not mutually independent.